文章编号：1009-6744 (2005) 05-00101-20

A Planning Model for Determining Optimal CMS Locations on a Freeway-Arterial Network

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Abstract: This paper presents an optimization model for locating Changeable Message Signs (CMS) on an integrated freeway-arterial network. Compared to existing models, the proposed model represents a well-balanced compromise between computational efficiency required to solve problems of realistic size, and model realism to ensure the quality of solutions. The model has three unique features: (1) it recognizes that locating CMS is a planning problem that must take into account both current and future needs and benefits; (2) it evaluates CMS benefits over multiple time periods with different traffic distributions; (3) it explicitly considers inherent variations in incident characteristics across links and over time. A sensitivity analysis is performed to examine the potential impacts on optimal CMS locations resulting from uncertainties in various input parameters such as traffic demand, incident attributes and driver behaviour. Lastly, the proposed model is applied to the Highway 401 express-collector freeway system in Toronto and CMS location solutions are examined.

Key words: changeable message; signs (CMS); location; optimization; traffic assignment; queuing theory

CLC number: U49

可变信息标志系统(CMS)选址优化模型

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摘要： 提出了一个在有高速公路和城市干道的组合路网中设置可变信息标志(CMS)组位置的最优化模型.与文献中现有的模型相比较,本文的模型有效地达到了实际规模道路网络计算复杂性问题和确保求解模型的质量之间很好的平衡. 本模型具有三个不同的特征:(1)认为在路网中设置安装可变信息标志(CMS)系统是一个必须同时考虑现在和未来需要和利益的典型的规划问题;(2)评估由于在路网中设置了可变信息标志(CMS)系统而带来的交通效益是通过具有不同交通分布型态的多个时段来完成的;(3)考虑了在不同的路段上和时段内交通事件的特征本质上是变化的因素.本文还进行了敏感性分析,用以检验在各种不同的交通输入参数中,如交通需求、交通事件特征值、以及驾驶员交通行为的不确定性对可变信息标志系统(CMS)在路网中的最优设置的潜在影响. 最后,本模型应用于加拿大多伦多的401 高速公路和辅道组成的混合道路系统,并且对CMS 系统设置结果进行了检验.

关键词： 可变信息;标志;选址;优化;路网;排队

中图分类号： U49

收稿日期:2005-04-18

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Introduction

Changeable Message Signs (CMS), also known as Variable Message Signs (VMS), are becoming popular as one of the primary means for transportation agencies to disseminate travel and traffic information to motorists. Under the umbrella of intelligent transportation systems (ITS), CMS constitute a key element in dynamic traffic management and information provision functions. CMS are commonly used to inform motorists of varying traffic, roadway, and environmental conditions and provide information on the location and severity of incidents and the expected delay. They can also be used to advise motorists of alternate routes in the event of an incident, construction or a roadway closure.

The effectiveness of CMS, however, depends on how many CMS are installed and where the CMS are located in the network. Theoretically speaking, the benefits from CMS could be maximized if the whole road network is instrumented with CMS. This is, however, practically impossible due to the high costs of installing CMS. Furthermore, past studies have suggested that excessive use of CMS could lead to diminishing returns in benefits, and even worse, could be counter-effective due to the behavioural response of drivers to real-time information (Wardman et al. 1997).

The planning of CMS locations is challenged by a variety of issues such as how to model the response of drivers to CMS messages, how to model random incidents that vary by time and space, how to model the impacts of incidents on traffic. The state-of-art practice has mostly relied on planners' experience and judgement, and is thus not the result of any comprehensive and systematic analysis. Abbas, et al. (1999) was the first in literature to study the problem of optimizing CMS locations in a road network. Their location optimization objective was to maximize the potential reduction in vehicle delay due to traffic diversion to alternative routes in response to incident information provided by CMS. A simple deterministic queuing model was used to estimate delays with and without CMS in a linear freeway network. However, it was not clear how issues such as congestion on alternative routes, over-saturated conditions, incident rates on individual links, and dependency of diversion rate on potential savings were handled in their model.

Another relevant study was initiated by Yi-Chang Chiu, et al. (2001), who proposed a bilevel stochastic integer programming model for the CMS location problem. The location optimization problem was realized at the upper level, seeking to maximize the total user benefit of real-time information from CMS. The users' responses (route choices) to incident conditions and information were represented at the lower level as a user optimal dynamic traffic assignment problem. The expected total user benefit corresponding to a given location solution was calculated based on a sample of benefits, each of which was estimated by generating a random incident on a network link and solving the resulting dynamic traffic assignment problem. The model also suffers from several limitations. First, as acknowledged by the authors, the whole process is extremely computationally intensive because of the need to evaluate a large number of candidate location plans, consider sufficient number of incident realizations for each location solution, and perform a simulation-based dynamic traffic assignment procedure for each incident realization. Second, their location benefit model was based on a route choice assumption that all users have perfect knowledge and real-time information on the network and incident conditions and possess the ability to anticipate other users' choice of routes and choose their optimal routes accordingly. Finally, it is unclear if it is practical or necessary to apply such a complex model, seemingly designed for operational management and control purposes, for solving the CMS location problem, which is essentially a planning problem.

In this paper, we propose a model that is aimed to strike a balance between computational efficiency and model realism. The major contribution of our research is in extending Abbas et al. 's work (1999) in three important aspects. First, we explicitly consider time-of-day variation in travel demand distribution by introducing a multi-period benefit estimation model. Second, we incorporate a logit route choice model in determining time-dependent division rate under incident conditions. Lastly, the proposed model explicitly takes into account inherent variations in incident characteristics across links and over time, such as incident rate, incident duration and capacity reduction. We only consider incident-induced delay in our benefit model since incidents are the major source of freeway delay (Shrank and Lomax, 2002). Additionally, the benefit of other CMS uses have not yet been quantified (e. g. fog/environmental information) or may be accomplished with portable signs or alternative devices (e. g. construction/maintenance/special events).

The paper is organized as follows. We first describe the individual components of the proposed model including: a time-dependent queuing model for estimating user delay with and without the presence of CMS, a dynamic traffic diversion model that relates the probability for a vehicle to divert from the incident link to its potential travel time savings, and a sequential optimization model for identifying the best locations for a given number of CMS. A sensitivity analysis then follows to examine the impacts of inherent variations in input parameters on the optimal CMS locations.

The Model

A time-dependent deterministic queuing model is used to estimate the benefit of a given set of changeable message sign (CMS) locations. The benefit is expressed as expected total reduction in user delay under incident conditions due to diversion in response to congestion and route guidance information displayed on CMS. Other types of CMS benefits from conditions such as recurrent congestion, scheduled lane/capacity reductions, and special events will not be considered. The expected delay is modeled as a function of many factors including network conditions, peak and off-peak demands, availability of alternative routes, route guidance information, compliance rates, etc. This section describes a systematic approach developed to predict the expected benefit from a given set of CMS and a methodology to optimize the locations for a given number of CMS.

Path-Based Traffic Assignment

In order to estimate the delay caused by incidents (with or without information), traffic volume estimates, for the time periods of interest, at individual network links must first be obtained. Typically, network traffic volumes are obtained using a link-based method (e. g. Frank-Wolfe method), which produces link flow estimates from a matrix of origin-destination flows based on Wardrop's user equilibrium (UE) assumption. This information is, however, not sufficient for the proposed benefit model as it requires not only the volume of traffic on a specific link but the individual path flows between origins and destinations of that traffic as well. This information is used both in the prediction of traffic diversion and the alternative route travel times. A conceptual difference between the link-based method and path-based assignment methods is illustrated in Figure 1.

Generally, there are two classes of path-based assignment methods: incremental and user-equilibrium. The incremental method performs successive all-or-nothing assignments on a parsed OD trip matrix (Sheffi, 1985). Incremental assignment does not guarantee to result in a UE condition and will not be considered further. Several user-equilibrium path-based methods for traffic assignment have been developed, including a modified Frank-Wolfe approach. However, the gradient projection (GP) method proposed by Jayakrishnan et. al. (1994) for traffic assignment has thus far proven the most efficient Chen et. al. (1999) and will be used here.

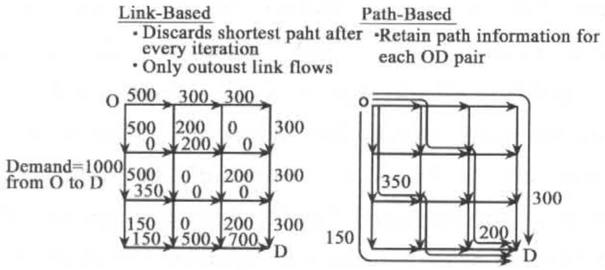

Fig. 1 Conceptual Differences Between Assignment Methods

Incident Delay Model without Information

A time-dependent deterministic queuing model is used to estimate user delay on a specific link during an incident. The time of day is divided into several time periods, depending on temporal variation of demand distribution. The total user delay for each of the time periods may be classified into one of three cases: Case I — off-peak, Case II — peak under-capacity, or Case III — peak over-capacity. The queuing diagrams for Cases I, II, and III are shown in Figure 2. The shaded area represents the total user delay when an incident occurs on the specific link a at a specific period p, denoted by D_a^p.

The incident occurrence time, denoted by t^o, is defined for each of the three cases as follows:

$$t^o = \begin{cases} 0 & \text{Case I} \\ [0 \to t^p] & \text{Case II, Case III} \end{cases} \quad (1)$$

Where t^p represents the end of the peak period under consideration, p, and start of the off-peak period $p+1$. Note that for the off-peak case (Case I), it is assumed that all links are under-saturated in normal traffic conditions and the incident occurrence time has negligible effect on the user delay. Therefore, to simplify calculations, only incidents that occur at the beginning of the time period will be considered ($t^o = 0$). It is additionally assumed that, for the peak period cases (Case II and III), the incident occurrence time is uniformly distributed from zero to the period duration under consideration. To determine a value for user delay for the peak period cases, a range of incident occurrence times must be considered and the user delay results averaged for each of these occurrence times. An arithmetic mean of user delays for a given number of incident occurrence times are evaluated. This same method is applied in the following section for estimating delay with information.

The cumulative arrivals and departures at the incident occurrence time (t^o), denoted by N_{arr}^o and N_{dep}^o respectively, can be calculated using Equations 2 and 3:

$$N_{arr}^o = x^p \cdot t^o \quad \text{Case I, Case II, Case III} \quad (2)$$

$$N_{dep}^o = \begin{cases} x^p \cdot t^o & \text{Case I, Case II} \\ S \cdot t^o & \text{Case III} \end{cases} \quad (3)$$

Where:

x^p — flow rate on specific link during normal traffic conditions (non-incident) during period p. The arrival (flow) rate is assumed to be known for each of the time periods (veh/hour);

S — maximum capacity of specific link (veh/hour);

t^o — incident occurrence time (hours).

Note that, for the under-capacity cases, the numbers of cumulative arrivals and departures are equivalent at the incident occurrence time. However, since the link is over capacity for Case III, the cumulative number of arrivals is greater than departures.

Similar to the occurrence time, the time of incident clearance (t^c) and the corresponding cumulative number of departures (N_{dep}^c) is defined for each of the three cases as follows:

$$t^c = t^o + \tau \quad \text{Case I, Case II, Case III} \quad (4)$$

$$N_{dep}^c = N_{dep}^o + C \cdot \tau \quad \text{Case I, Case II, Case III} \quad (5)$$

Where:

τ — time required to clear the incident (hours);

C — reduced capacity of the link during the incident (veh/hour). Note that case-specific values could be used for C;

t^o — incident occurrence time (hours);

N^o_{dep} — cumulative number of departures at the incident occurrence time.

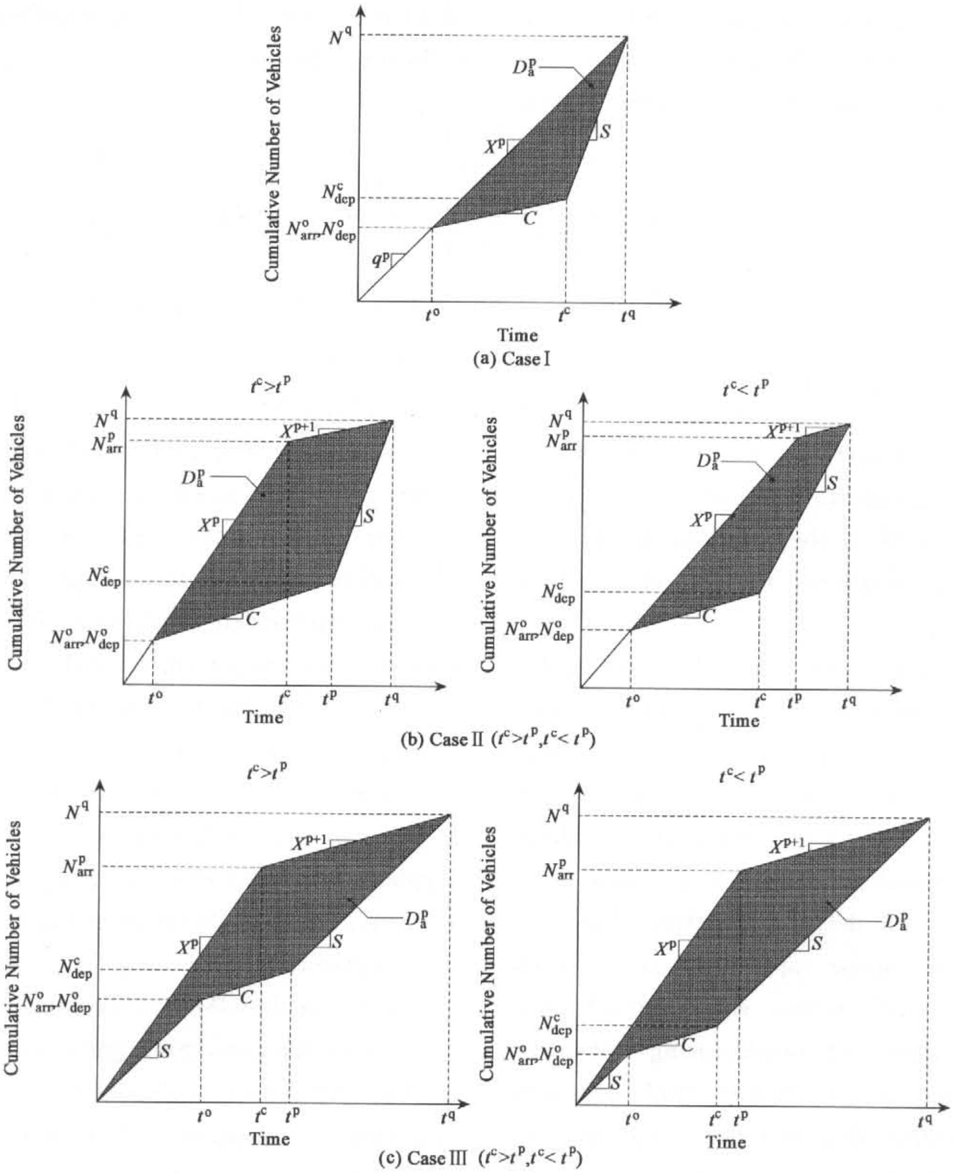

Fig. 2 Vehicle Queuing Diagram Under Incident Conditions

Note that the cumulative number of arrivals at the time of incident clearance is not required to determine the user delay for any of the cases.

Recalling the assumption that the original arrival rate for a specific link (x^p) and the duration of each time period (t^p) are both known. Therefore, the cumulative number of arrivals at the end of the peak period p under consideration, denoted by N^p_{arr}, can be determined from the following equation:

$$N^p_{arr} = x^p \cdot t^p \qquad \text{Case II, Case III} \tag{6}$$

Similar to the cumulative number of arrivals at the time of incident clearance, the cumulative number of departures at t^p is not required to determine the user delay for any of the cases. Also recall that for the off-peak incident case, Case I, the incident is assumed to have occurred at the start of the time period under consideration. Therefore, for Case I under normal conditions, the end of the time period will occur after the incident queue clearance time. If the time required to clear the incident queue is long and/or the accident occurrence time is close to the end of the period, the queue will extend to the next period. Therefore the uniform arrival assumed in Case I is no longer valid and therefore Case II should be used.

The incident queue clearance time (t^q) and the cumulative number of vehicles at the queue clearance time (N^q) are determined from the following equations:

$$t^q = \begin{cases} \dfrac{(S-C)\cdot t^c}{(S-x^p)} & \text{Case I} \\ \dfrac{x^p\cdot(t^p-t^o)-x^{p+1}\cdot t^p+(S-C)\cdot t^c+C\cdot t^o}{(S-x^{p+1})} & \text{Case II} \\ \dfrac{(x^p-x^{p+1})\cdot t^p+(S-C)\cdot(t^c-t^o)}{(S-x^{p+1})} & \text{Case III} \end{cases} \quad (7)$$

$$N^q = \begin{cases} x^p\cdot t^q & \text{Case I} \\ x^p\cdot t^p + x^{p+1}\cdot(t^q-t^p) & \text{Case II, Case III} \end{cases} \quad (8)$$

Refer to Equations 1 — 6 for further explanation of parameters. Furthermore, using Equations 1—8, all of the points of the queuing diagram can be determined for each of the three cases.

It is important to note that the delay estimation methodology discussed above, while commonly used in literature such as Highway Capacity Manual (2000), does not account for queue spill-back and its possible effects on delay estimation. Queue spill-back may cause three possible effects on delay estimation. The first effect is that queue spill-back may block neighbouring intersections, which would then reduce the capacity of neighbouring links and cause additional delays. Queue spill-back may also induce traffic diversion from the incident link, which would lead to an arrival rate at the incident link lower than what would normally be expected under non-incident conditions. Lastly, queue spill-back may block diversion access points such as freeway off ramps, preventing drivers from alternating their routes. These effects are difficult to accurately represent in a delay estimation model, requiring detailed knowledge of network geometry and driver behaviour.

Diversion Model

To be able to quantify the benefits of a given CMS, a diversion model is required to predict the number of vehicles that would divert to alternative routes due to message activation of the CMS during incident conditions. When a driver is provided information from a CMS that an incident has occurred along the intended travel path, a decision is made to either stay on the same route or divert to an alternative route. This decision depends on various factors such as severity of the incident, current extent of queue caused by the incident, the driver's experience and familiarity of the network, and incident characteristics delivered via the CMS. Therefore, modeling the underlying decisions is a significant challenge due to the behavioural complexity of the drivers' response to incidents and incident information (Wardman et al. 1997). In the two existing studies on the CMS location problem, Abbas et al. (1999) assumed a constant fixed diversion rate regardless of availability of alternative routes, severity of incidents and various other factors. In Yi-Chang Chiu et al.'s simulation-based model (2001), a bounded route choice model was applied, assuming a driver would divert to an alternative route if the expected travel time saving exceeds a certain threshold.

In this study, we propose to use a simplified discrete choice model to capture the major characteristics of drivers' common response behaviour under incident conditions. The model was motivated by the empirical work of Wardman et al. (1997), assuming that the probability for a driver to choose to divert depends on the expected travel time saving from diverting with the following logit form:

$$P_{k,m}(t) = \frac{1}{1 + e^{\alpha - \beta \cdot S_{k,m}(t)}} \tag{9}$$

Where:

$P_{k,m}(t)$ — probability for a vehicle, arriving at time t and traveling through CMS k on path m, to divert to an alternative route;

$S_{k,m}(t)$ — travel time savings ratio defined in Equation 10 as the expected savings of using the alternative route divided by the travel time of the alternative route;

α, β — model parameters.

The travel time savings ratio, $S_{k,m}(t)$, is based on the expected delay that vehicles joining the incident queue will experience and the travel time through the shortest alternative route, as defined as follows:

$$S_{k,m}(t) = \frac{T_{k,m}(t) - T^*_{k,m}}{T^*_{k,m}} \tag{10}$$

Where:

$T_{k,m}(t)$ — expected travel time a vehicle joining the queue will experience at time t (hours);

$T^*_{k,m}$ — travel time of the shortest alternative route, not traversing the incident link, from CMS k to the destination node of path m (hours).

The expected travel time experienced by a vehicle joining the queue, $T_{k,m}(t)$, is based on two components. The first component is the expected travel time from the CMS link, through the incident link, to the destination node of the path under incident-free conditions. The second component is based on the expected queuing delay for a vehicle if it were to continue on its original path.

The diversion model represented by Equation 9 suggests that the proportion of vehicles that would divert increases as the travel-time savings increase. The relationship between diversion probability, or rate, and travel time savings is depicted in Figure 3 under three assumed combinations of model parameters. The curves developed based on the logit model structure are intuitively correct: the higher the travel time savings, the higher the probability for a vehicle to divert; drivers are usually reluctant to change routes with a small percentage of savings. For example, for the case of $\alpha = 5$ and $\beta = 5$, when an alternative route is estimated to provide 50% travel time savings, there would be 50% chance for the driver to make a diversion.

The parameters α and β are essentially to model variations indrivers' characteristics (e.g., aggressiveness) and information attributes (e.g., types, level of reliability, frequency etc.). Realistic estimates of these model parameters could be obtained through a statistical analysis of stated preference or revealed preference survey results as in Wardman et al. (1997). A sensitivity analysis is performed in this study to evaluate the potential impact of these parameters on the final CMS location solutions.

Arrival time and reduced flow rate due to a CMS may be determined by applying Equations 9 and 10, derived for a single path, to all paths, as follows:

$$\hat{x}_k(t) = \sum_m f_{k,m}[1 - P_{k,m}(t)] \tag{11}$$

Where:

$\hat{x}_k(t)$ — flow on the incident link, or reduced arrival rate, for vehicles that traverse both CMS k and the incident link (veh/hour);

$f_{k,m}$ — flow on the mthpath passing both CMS k and the incident link (veh/hour);

$P_{k,m}(t)$ — probability for a vehicle traveling through CMS k, on pathm, to divert to an alternative route, as defined in Equation 9.

In determining the reduced arrival rate at a given link due to all CMS, it is assumed that drivers would defer their decision until they reach the CMS that is closest to the incident link. That is, for a path flow that traverses several CMS, only the CMS closest to the incident link has an effect on the diversion rate. An activating zone is also considered so that only CMS within a certain distance of the incident will display information.

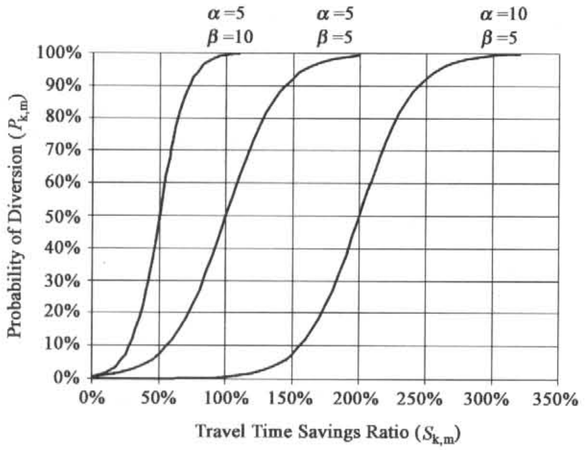

Fig. 3 Diversion vs. Travel Time Savings

Incident Delay Model with Information

The time-dependent deterministic queuing model may be combined with the diversion model to estimate the reduction in user delay for a given incident link and a given set of CMS. The set of CMS is ordered based on the shortest path travel time from the CMS link to the incident link, T_k, with the shortest time at the start of the CMS set and the longest time at the end of the CMS set. The method of determining the reduction in incident delay may be best illustrated using an example based on the simplest off-peak case, Case I. Consider any network with three CMS, ordered from the closest to longest from the incident link, and an opportunity to divert to an alternative route after each of the CMS. The corresponding queuing diagram for this simple network is shown in Figure 4. Although this method is illustrated with three CMS, the extension to any number of CMS is straightforward.

Shown in Figure 3, an incident occurs at time t^o, and, after a time lag including three major components: incident detection time, information processing time, and CMS activating time, diversion due to CMS starts at time t^s. The incident link does not experience a reduction in arrival rate until $t^s + T_1$ due to the travel time component from the firstCMS to the incident link. The same is true for additional reduction in arrivals caused by CMS 2 and CMS 3.

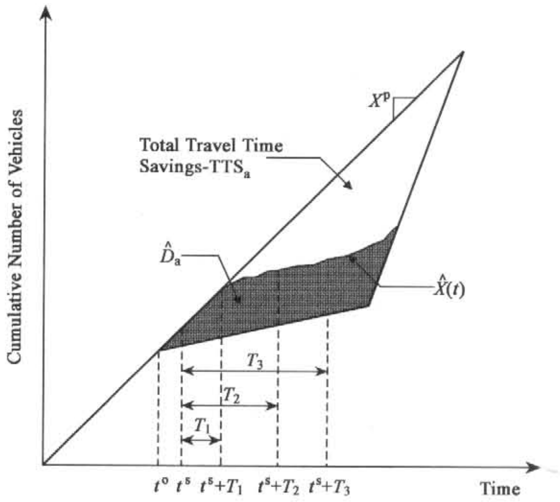

Fig. 4 Simple Network Queuing Diagram

Determining the actual reduced flow rate at each interval between the diversion start times corresponding to two neighbouring CMS is not straightforward. This is because the reduction in flow rate depends on the proportion of traffic diverted due to CMS information, while traffic diversion is a function of travel time savings, which in turn depends on how much traffic is diverted. We solve this interdependency problem by constructing the reduced flow rate curve from left, starting at $t^s + T_1$, to right at a small time interval (e. g. 5 minutes). At the start of each interval, the cumulative number of arrivals is calculated based on the reduced arrival rate of the previous interval. This cumulative arrivals is then used to estimate the expected delay for a vehicle arriving during this time instance and all vehicles arriving at the current interval are assumed to experience this same delay. The diversion rate can then be estimated for the corresponding interval. This process continues until the reduced arrival curve, $\hat{x}(t)$, intersects with the cumulative departure curve.

Another aspect of the benefit estimation model is the incident rate for a link and the consideration of incidents for different time periods. In order to determine the expected daily user delay, the following equation is used to estimate the expected number of incidents that would occur on link a over a specific period p, (n_a^p).

$$n_a^p = \Delta^p \cdot x^p \cdot L_a \cdot r_a \tag{12}$$

Where:

Δ^p — duration of time period p, which is assumed to be known;

x^p — original link arrival rate (veh/hour);

L_a — length of link a (km);

r_a — incident rate for link a (# incidents/million-veh-km).

The benefit estimation model can now be completely defined to estimate the expected travel time savings for a specific link a during time period p (TTS_a^p) as follows:

$$TTS_a^p = (D_a^p - \hat{D}_a^p) \cdot n_a^p \tag{13}$$

Where:

D_a^p — the total vehicle delay on link a, during time period p, without CMS information (veh-hours);

$\hat{D}_a^p$ — the total vehicle delay on link a, during time period p, with CMS information (veh-hours);

n_a^p — incident rate for link a, during time period p, as defined in Equation 12 (# incidents).

With this benefit model we can now define the CMS location problem as follows: identify the location of a given number of CMS in a road network so that the following objective function for travel time savings over all p time periods and all incident links, denoted by TTS, is maximized.

$$\text{Max } TTS = \sum_p \sum_a TTS_a^p \tag{14}$$

Where:

TTS_a^p — travel time savings as defined in Equation 13

Effect of Diversion on Alternative Routes

The diversion model presented approximates the rate at which drivers divert from their originally intended route to one or more alternatives during incident conditions. However, no prediction is made as to what those alternative routes are, since there are presently no reliable route choice models during traffic equilibrium disruption. Additionally, the increased travel time experienced by drivers on these alternatives are not considered in the travel time savings calculation of Equation 13.

To partially account for this impact an alteration to the diversion rate and incident link-based travel time savings is proposed that re-assigns diverted traffic to the shortest alternative route between the CMS link and the trip destination. A diversion equilibrium is approximated through successive iterations of $T_{k,m}(t)$ and $T_{k,m}^*$ in Equation 9. During the iterations, link travel times are updated to reflect volume changes resulting from traffic diversion. These updated link travel times may then be included in Equation 14 to adequately reflect the negative impact of diverted traffic on otherwise unaffected links.

$$\text{Max } TTS = \sum_p \sum_a (TTS_a^p - TTI_a^p) \tag{15}$$

Where:

TTS_a^p — travel time savings as defined in Equation 13.

TTI_a^p — travel time increase on alternate routes caused by incident on link a during time period p.

The Solution Procedure

A greedy CMS allocation algorithm is used to optimize CMS locations. The greedy method was chosen because it most closely resembles the administrative decision-making process that incrementally adds changeable message signs to a road network. The incremental method was used for traffic assignment to identify path flows. This method was observed to produce link volumes close to that of the *F-W* equilibrium traffic assignment and a path-based traffic assignment procedure for the sample network used in the following section. The overall solution process is shown in Figure 5, which has been implemented in a software tool called OptimalCMS and used in the following sensitivity analysis.

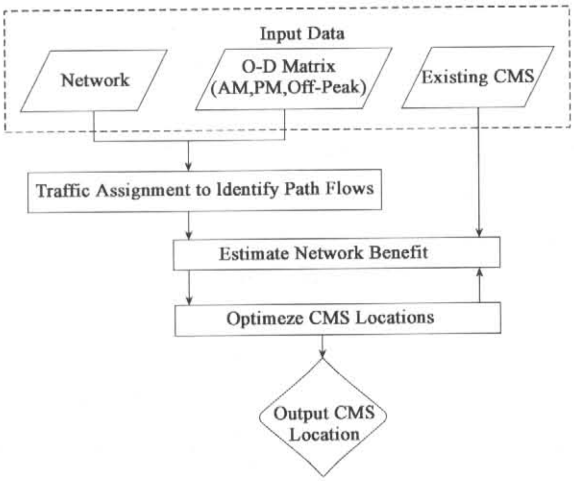

Fig. 5 A framework for locating

Sensitivity Analysis

The proposed model for evaluating the expected reduction in delay, due to the installation of CMS, is deterministic in nature, assuming perfect information on input parameters such as O-D demand, incident conditions and traffic diversion behaviour. In practice, however, most of these parameters are inherently uncertain due to various factors, such as insufficient relevant data to estimate the model parameters, errors in both the raw data and model specification, and errors in predicting future traffic demand and conditions. The objective of this section is to quantify the potential effects of these variations on CMS location. These effects will be explored using both a hypothetical network (Case A) and a section of Highway 401 in Toronto (Case B).

Case A: Sample Network

Figure 6 shows the layout of one of the road networks used in our sensitivity analysis, which is a modified version of a sample network used in the Integration simulation model (Van Aerde, 2000). The network represents a freeway-arterial system, consisting of 56 nodes and 124 links. An east-west freeway is located in the middle of the network surrounded by arterial links with a total length of 6.4 km.

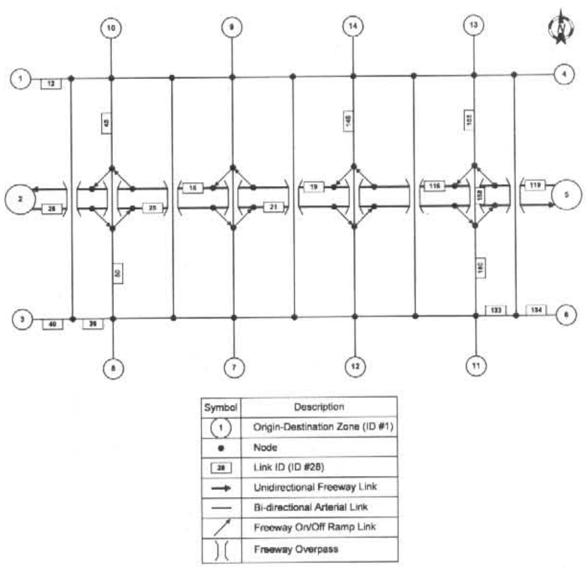

Fig. 6 Sample Network

Also shown in the network are 14 origin-destination zones as trip generators. Four time periods are considered: AM peak, midday off-peak, PM peak, and overnight off-peak with period durations of 2, 7, 3, and 12 hours respectively. The origin-destination (O-D) demand is given in Table 1. Generally, a large number of trips travel eastbound (from west to east) during the AM peak period and, conversely, a large number of trips travel westbound (from east to west) during the PM peak period. The rate of AM demand is monotonically higher than the PM demand. Also, the demand matrix for modelling uncertainty in traffic demand and driver behaviour is different (larger demand) than the demand matrix for modelling uncertainty in incident attributes.

Table 1 Base Daily O-D Demand Matrix (vph)

ID	FromZone	ToZone	AMPeakTripRate	MiddayTripRate	PMPeakTripRate	OvernightTripRate
1	2	4	2000	100	100	50
2	2	6	2000	100	100	50
3	2	5	4000	100	100	50
4	5	2	100	100	1500	50
5	5	1	100	100	3000	50
6	5	3	100	100	1500	50
11	4	1	100	100	100	50
12	4	2	100	100	1000	50
13	4	3	100	100	100	50
16	1	4	100	100	100	50
17	1	5	2000	100	100	50
18	1	6	100	100	100	50
26	3	4	100	100	100	50
27	3	5	2000	100	100	50
28	3	6	100	100	100	50
32	6	1	100	100	100	50
39	6	2	100	100	2000	50
40	6	3	100	100	100	50
41	10	8	100	100	100	50
42	10	7	100	100	100	50
43	10	11	100	100	100	50
44	10	12	100	100	100	50
45	9	8	100	100	100	50
46	9	7	100	100	100	50
47	9	11	100	100	100	50
48	9	12	100	100	100	50
49	13	8	100	100	100	50
50	13	7	100	100	100	50
51	13	11	100	100	100	50
52	13	12	100	100	100	50
53	14	8	100	100	100	50
54	14	7	100	100	100	50
55	14	11	100	100	100	50
56	14	12	100	100	100	50
57	7	9	100	100	100	50
58	7	10	100	100	100	50
59	7	13	100	100	100	50
60	7	14	100	100	100	50
61	8	9	100	100	100	50
62	8	10	100	100	100	50
63	8	13	100	100	100	50
64	8	14	100	100	100	50
65	11	9	100	100	100	50
66	11	10	100	100	100	50
67	1	11	100	100	100	50
68	1	12	100	100	100	50
69	4	8	100	100	100	50
70	4	7	100	100	100	50
71	4	11	100	100	100	50
72	4	12	100	100	100	50

The base incident rate was set at 2.9 incidents per million-veh-km according to a report from National Highway Traffic Safety Administration (2000). Equation 12 was used to convert this value, r_a, to incidents over a given time period, n_a^p. Other base values used for this sample network and applied to all incident links are: capacity reduction of 0.8, incident duration of 30 min, detection time of 10 min, and processing and CMS activating time of 5 min. These values are similar to incident characteristics used in other relevant literature (Abbas et al., 1999 and Chiu et al. 2001). Impact of variations in these parameters on optimal CMS location are also analysed in this section.

Comparison to Heuristic Methodology

In the absence of a methodical approach for optimizing CMS locations, traffic managers usually locate CMS in a heuristic ad-hoc way based on freeway link traffic volume and diversion opportunities at downstream off-ramps. To illustrate the difference between the proposed model and this ad-hoc approach, we consider the greedy allocation results for the base case of the sample network. Shown in Table 2 is the order of CMS allocation based on the proposed model, and, the revised order based on link traffic volume. It is reasonable to assume that there is a good opportunity for traffic to divert to an alternative route from each of these links since they were all selected in the greedy allocation process.

Table 2 CMS Allocation Order, Proposed Model vs. Link Volume

Allocated CMS at Greedy Iteration #	Link ID (Base Case)	Daily (veh)	CMS Order (Volume)
1	119	22 500	6
2	28	21 295	7
3	116	53 310	1
4	25	47 930	2
5	155	20 345	8
6	45	17 980	9
7	50	14 350	10
8	158	24 105	5
9	16	47 790	3
10	21	46 750	4

As seen in Table 2, there is a significant difference between CMS allocationbased on traffic volume, and, allocation based on the proposed model. Using the proposed model, freeway links with highest volume are not necessarily the first assigned with a CMS.

Optimal Number of CMS

Figure 7 shows the relationship between the total benefits of CMS and the total number of CMS installed in the network. A significant increase in the total network benefit resulted from the addition of the first 3 CMS to the network (an increased travel time saving of about 62% from 1 CMS to 2 CMS). The marginal network benefits from adding more CMS tends to become less significant and level off after the third CMS is added. This result is expected due to the fact that when the coverage of CMS reaches to a certain level, majority of the O-D flows will be covered by the CMS, thus the benefits of adding more CMS become small. This general phenomenon of diminishing returns is commonly seen in economics and suggests the potential of obtaining the optimal number of CMS for a given network by trading off the installation costs and the resulting benefits.

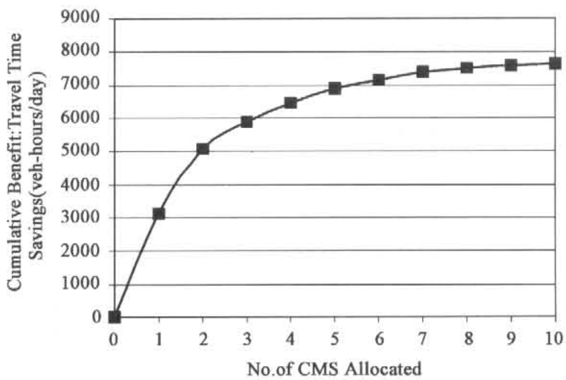

Fig. 7 Marginal Benefit of CMS Allocation

Consideration of Multiple Time Periods

Most existing models for optimizing CMS locations consider only a single time period, AM or PM peak, and ignore the traffic exposure for the remainder of the day. The proposed model, however, is able to consider the entire day in the optimization process. The CMS allocation results based on a single time period compared to the results for all time periods are shown in Table 3.

It is evident that only considering a single time period in the optimization process does not produce the best CMS locations since demand is directional and the highest volume freeway links during the AM peak period are not the highest volume freeway links during the PM peak period. Also, incidents during the off-peak period had a negligible impact on the final location solutions of the optimization process as the estimated benefit was several orders of magnitude smaller than the benefit predicted during the peak periods.

Table 3 Greedy Allocation Results Based On Time Period

Allocated CMS at Greedy Iteration #	All Periods		AM Peak		Midday Off-Peak		PM Peak		Overnight Off-Peak	
	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *
1	119	3123	28	1941	45	3	119	3120	16	0
2	28	1942	25	556	16	2	116	841	45	0
3	116	849	45	223	155	1	133	407	125	0
4	25	560	39	142	50	1	155	266	155	0
5	155	406	21	55	160	1	19	63	50	0
6	45	268	145	30	21	0	176	28	160	0
7	50	224	75	29	116	0	55	20	105	0
8	158	142	125	9	125	0	58	14	174	0
9	16	64	12	7	25	0	16	9	68	0
10	21	55	35	4	106	0	145	5	80	0

* Marginal Savings (Veh—hours/day)

Uncertainty in Traffic Demand

The uncertainty in traffic demand was modelled by considering random fluctuations in the origin-destination matrix. Variations of +10%, −10%, ±5%, ±10%, and ±20% to the base origin-destination demand matrix were evaluated. The variation in each case was developed by increasing or decreasing each entry in the base O-D matrix (AM, PM, and off-peak) by a certain amount. For example, for the case of + 10% variation, each demand entry was increased by a random amount from 0% to 10% inclusive. The − 10% variation in demand was determined in a similar manner. The ± 5%, ±10%, and ± 20% variations in demand were also calculated similar to the + 10% variation with the exception of the interval of equal likelihood, which is increased from 0%→10% to − 10%→ 10% for the ± 10% variation. The results of the greedy allocation procedure for each of the demand variations are shown in Table 4.

Table 4 CMS Allocation Results for Variable Congestion Levels

Allocated CMS at Greedy Iteration #	Base Case		+10%		−10%		−5%～+5%		−10%～+10%		−20%～+20%	
	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *
1	119	3123	119	3489	119	2997	119	3254	119	3628	119	2448
2	28	1942	28	2087	28	1675	28	1921	28	1918	28	2123
3	116	849	116	933	116	812	116	865	116	891	116	772
4	25	560	25	601	25	508	25	562	25	563	25	575
5	133	406	133	449	133	372	133	422	133	415	133	331
6	155	268	155	308	155	242	155	279	155	312	45	248
7	45	224	45	240	45	197	45	221	45	223	155	222
8	39	142	50	144	50	134	39	147	39	144	50	129
9	19	64	21	72	19	57	19	58	19	63	21	70
10	21	55	19	66	21	51	21	55	21	55	19	63

* Marginal Saving(veh-hour/day)

As seen in Table 4, variations in demand do not significantly effect the allocation of changeable message sign locations. A slight variation in CMS allocation order was observed, but this was only for a few locations with marginal travel time savings benefit.

Uncertainty in Incident Conditions

Four incident attributes, including incident rate, incident duration, incident occurrence time, and capacity reduction, were modelled for uncertainty. Since the incident occurrence time, as mentioned earlier, is assumed to be distributed uniformly over the time period of concern it is not considered hereafter. The uncertainty in incident rate was modelled by considering random fluctuations in the link exposure-based incident rate. Fluctuations of +10%, −10%, +50%, −50%, +100%, and −100% were considered and developed as previously described for other variations. The CMS location results for variations in link incident rates are shown in Table 5.

Table 5 CMS Allocation Results for Variable Link Incident Rates

Allocated CMS at Greedy Iteration #	Base Case		+10%		−10%		+5%		−5%		+100%		−100%	
	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *	Link ID	Marginal Savings *
1	119	559	119	545	119	573	119	735	119	549	119	605	119	283
2	28	417	28	444	28	432	28	375	28	381	28	461	28	218
3	116	138	116	134	116	138	116	173	116	114	116	160	155	56
4	25	103	25	107	25	106	25	94	25	83	25	116	45	41
5	155	49	155	48	155	51	155	65	155	62	45	50	116	33
6	45	43	45	46	45	44	45	39	45	44	155	44	50	29
7	50	25	50	27	50	26	158	33	158	31	50	26	158	19
8	158	24	158	24	158	25	50	22	50	26	160	22	21	17
9	16	15	21	15	16	16	19	18	16	13	16	22	25	4
10	21	15	16	15	21	16	21	16	21	12	21	17	19	3

* Marginal Savings(veh-hours/day)

The optimal CMS locations are insensitive to smaller variations of 10% and 50% in the link incident rate, however, larger variations seem to affect the allocated CMS locations. The reason for the effect on CMS locations is the linear relationship between incident rates and travel time savings, the increase in travel time savings is directly proportional to the increase in link incident rates. The link exposure-based incident rate variable may be stochastic as opposed to deterministic as was assumed.

An 80% reduction in link capacity was used for the base case. Four additional cases were generated: the first two cases involved monotonic reductions of 40% and 60% for all links; the other two cases had variations similar to the random fluctuations in congestion level. A random decrease in capacity with equal likelihood of decrease from 50% to 70% inclusive for one case, and 40% to 80% for the other case were generated independently for each of the links. The CMS location results for capacity reduction variations were identical for the first six CMS allocated and only the order of allocation for the last four varied from the base case. This indicates that the optimal CMS location is insensitive to uniform changes in capacity reduction and relatively insensitive to random fluctuations in capacity reduction. Again, a slight variation in CMS allocation order was observed but only for a few locations that produced a small marginal benefit.

Variations in incident duration were evaluated using 30 minutes for the base incident duration case, and uniform durations of 40 minutes and 50 minutes for additional uniform incident duration cases. Random fluctuations, similar to those discussed for capacity reduction, were modelled using two additional random duration cases with equal likelihood for durations between 30 minutes and 50 minutes inclusive for the first random case and between 20 minutes and 60 minutes inclusive for the second random case. The results of the allocation procedure, similar to those of variations in link capacity reduction, show identical CMS locations for the first six allocated and differing order for the last four allocated. Therefore, with the exception of a few locations with marginal benefit, the allocated CMS location is insensitive to both monotonic and random variations in incident duration.

Case B: Toronto Network

The Toronto case will now be considered for additional analysis. The complete dataset (Figure 8) provided by the Ministry of Transportation of Ontario, consists of 14160 nodes, 37386 links, and 69448 O-D pairs extracted from the emme/2 transportation planning software. This study area is prohibitively large for model execution, therefore, a smaller network (Figure 9) mainly comprised of Highway 401 and its nearby arterials was extracted for computational analysis. This reduced the number of nodes, links, and O-D pairs to 961, 2363, and 6149 respectively. The longest of these links being 3.45 km and the shortest being 0.04 km. Also, the link free-flow speeds range from 40 ~ 70 km/h for the arterials and ramps and 110 km/h for the freeway links. The smaller study area is approximately 23.2 km in the east-west direction, from west of Highway 410 to east of Allen Road, and 17.4 km from the most southerly node to the most northerly node.

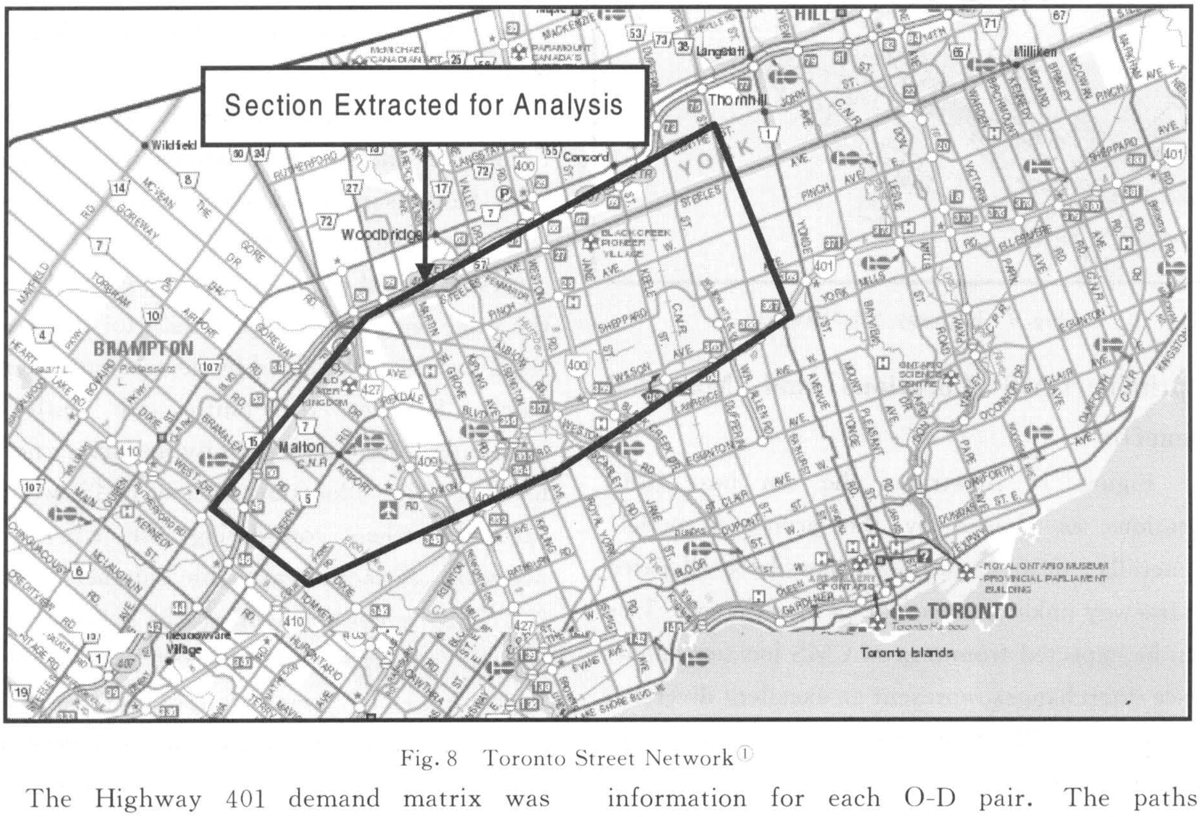

Fig. 8 Toronto Street Network①

The Highway 401 demand matrix was determined by performing path-based assignment for the complete dataset and saving the path information for each O-D pair. The paths, referenced as a succession of nodes, were truncated at the edges of the study area. The

① MTO Official Road Map: http://www.mto.gov.on.ca/english/traveller/map/images/pdf/southont/enlargements/Toronto.pdf truncation was performed by eliminating all nodes up to the first node that was part of the smaller network, this node being the origin node, then continuing until the last node that appears in the network is found, this node being the complementary destination node. The demand for this new O-D pair was then set to the associated path flow. Demands with common origins and destinations were combined to reduce the size of the derived demand matrix. The peak hour demand (AM) for this matrix is approximately 152,000 vehicles and the associated daily demand is approximately 948 500 vehicles.

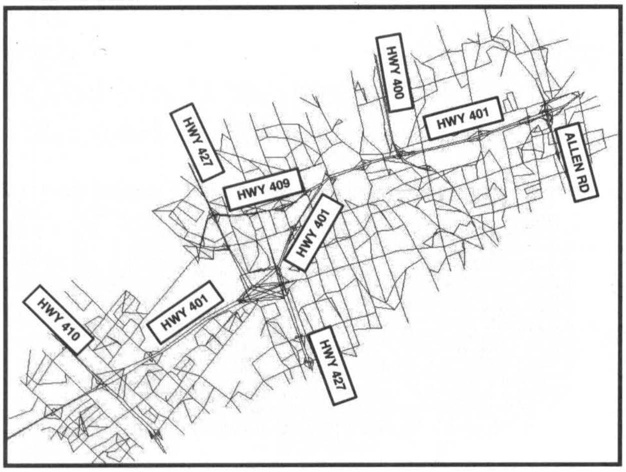

Fig. 9 Highway 401 Network

Optimal CMS Locations and Marginal Benefits

Figure 10 illustrates the ten best CMS locations as chosen by the proposed model. Generally, most of the CMS have been allocated to freeway links upstream of an interchange. This can be expected from a good CMS location model since interchanges represent an excellent diversion opportunity. The first five of these locations are described below.

• Link 7628 is located at the easterly end of westbound Highway 401. This location captures most of the westbound freeway traffic while providing a diversion opportunity to Allen Road.

• Link 9980 is located on westbound Highway 401 just after the on-ramp from southbound Highway 400. Highway 400 as well as Highway 401 traffic pass this point and can be informed of conditions on the Highway 401 main route and Highway 409→Highway 427 alternative route.

• Link 7547 is located on the eastbound Highway 401 after the Highway 427 interchange. The Highway 401 and 427 traffic may divert at a minor interchange downstream of the CMS link.

• Link 9986, located on eastbound Highway 401 at the Highway 400 interchange, has a high traffic volume but poorer diversion opportunities than the first three CMS allocated.

• Link 29 486, located at the beginning of the eastbound Highway 409, provides two excellent diversion opportunities through eastbound Highway 409 or southbound Highway 427. However, the traffic volume is much lower than the first four locations that CMS were located.

Similar to Figure 7 for the small network, Figure 11 shows the relationship between the total benefits of CMS and the total number of CMS installed in the GTA network for various diversion model parameters. The general trend is the same with a very high benefit for the first few CMS installed, followed by decreasingly marginal gains. Between the ninth and tenth CMS installation there is practically no further network benefit. The allocation of ten CMS would be excessive when considering the size of this network. A more reasonable number would be five CMS when considering the relatively small positive impact of CMS 6→10 and the limited opportunities to divert through interchanges.

The benefit of adding CMS to Highway 401 is approximately 1. 52 million veh-hours/year when considering an α and β of 5. This benefitis the difference between the incident-induced delay without CMS information (4. 44 million veh-hours/year) and the incident-induced delay in the presence of CMS information (2. 92 million veh-hours/year).

The network benefit may be converted to a dollar value by assuming a value of time, e. g. \$10/hour. Utilizing this value of time the benefit attained by installing one CMS, approximately 640 000 → 1 010 000 veh-hours, may be converted to a range of \$6.4 → \$10.1 million. After the installation of the second CMS the benefit increase to the range of \$8.5 → \$11.9 million. Again, a cost-benefit analysis would be complementary in determining the optimal number of CMS to install.

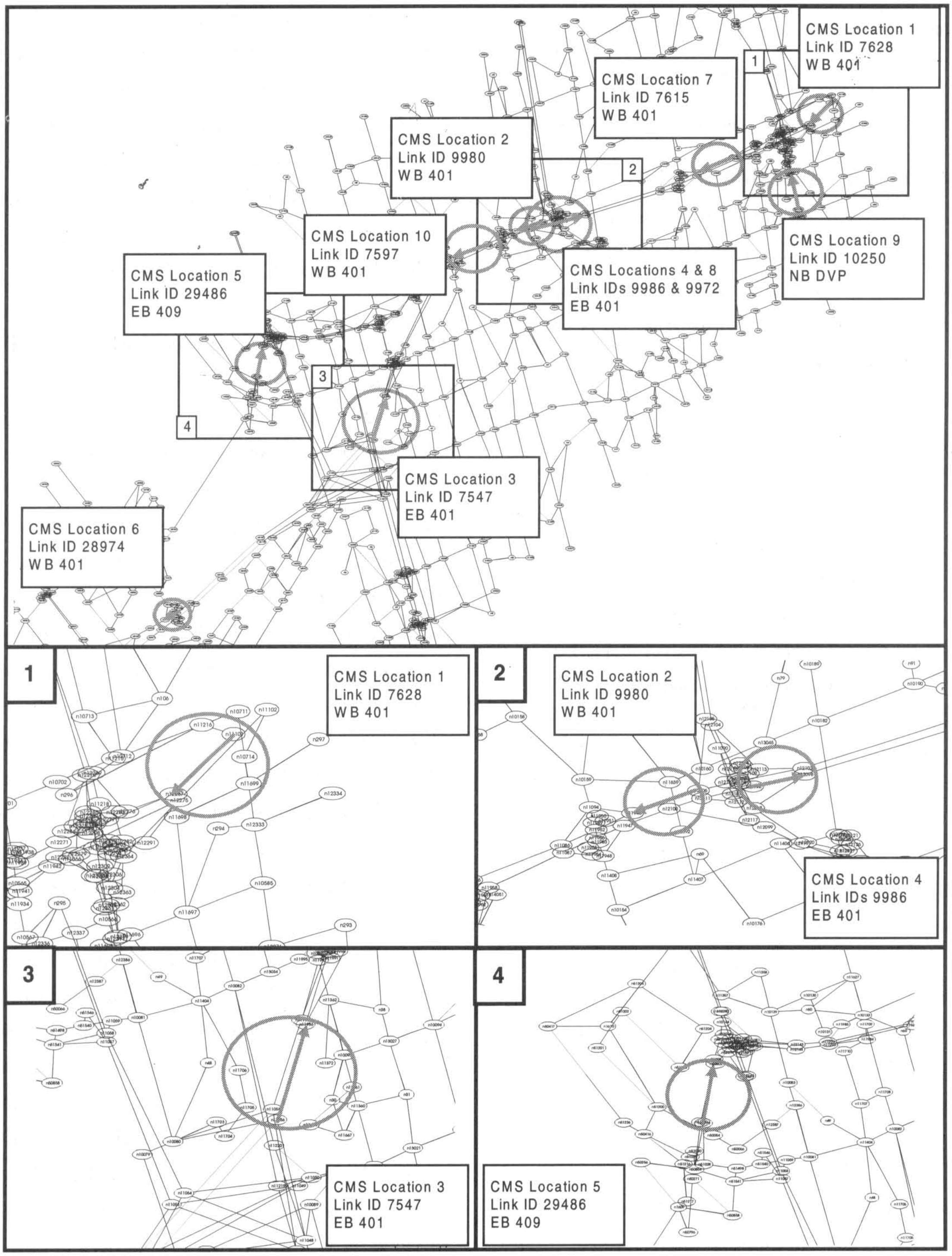

Fig. 10 Model Selected Optimal CMS Locations

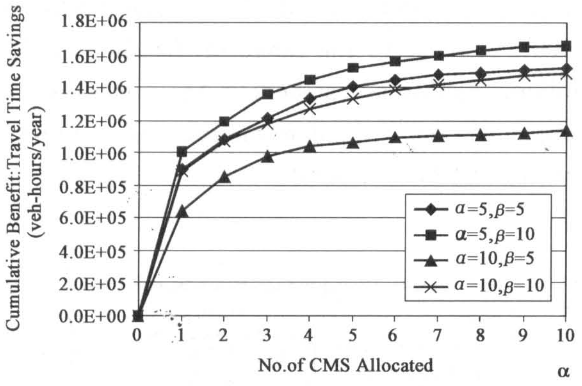

Fig. 11 Benefit vs. Number of CMS Installed

Uncertainty in Diversion Model

The diversion model contains a high degree of uncertainty. To identify the impact of the diversion model on CMS allocation, CMS were allocated on the basis of different α and β values. Recall from Equation 9 that increasing α will monotonically decrease the diversion rate for all paths and increasing β will increase the diversion rate, depending on the travel time for the shortest alternative route. Therefore, a comparison of the marginal benefits of each allocation procedure is not relevant. Instead the focus will be on a comparison of the chosen locations (Table 6).

Table 6 CMS Allocation Results for Variations in Diversion Model Parameters

Allocated CMS at Greedy Iteration #	$\alpha=5$ $\beta=5$ Link ID	$\alpha=5$ $\beta=10$ Link ID	$\alpha=10$ $\beta=5$ Link ID	$\alpha=10$ $\beta=10$ Link ID
1	7 628	7 628	7 628	7 628
2	9 980	7 547	29 007	9 980
3	7 547	9 980	7 593	7 547
4	9 986	9 986	7 607	9 986
5	29 486	29 469	9 969	29 486
6	28 974	28 974	7 615	28 974
7	7 615	10 250	10 250	9 774
8	9 972	7 559	10 089	10 421
9	10 250	7 615	28 039	7 575
10	7 597	7 597	25 008	9 966

As seen in Table 6, the CMS locations are highly sensitive to changes in thediversion model parameters. Generally, as β increases the actual travel time savings ratio, $S_{k,m}$, is less significant because $\alpha - \beta \cdot S_{k,m}$ becomes very large in the negative direction and $e^{\alpha-\beta \cdot S_{k,m}}$ approaches zero, so more and more incident links have close to 100% diversion. Since the savings ratio becomes less significant, the volume and number of incident links for which the CMS is effective becomes the predominant factor in locating the CMS. Therefore, as β increases the CMS are located closer to the start of the paths to maximize the number of links and traffic volume for which they are effective. Also, it was observed that these CMS links have longer alternative routes and lower travel time savings ratios compared to CMS links allocated at lower values of β.

Inclusion of Alternative Path Travel Time

As previously indicated, the increased delay incurred on alternative routes may be a factor in CMS location decisions. Therefore, a comparison was made between chosen CMS locations with and without the inclusion of the negative impact diversion has on alternative route travel times. Recall that both methods include the travel times on alternate routes, but the consideration of alternate path travel times are used along with diverted traffic information in determining diversion rates. As seen in Figure 12 the inclusion of alternate path travel time slightly increases the travel time benefit. This can be expected since the iterative approach to determining diversion rates would prevent excessive re-routing of traffic to congested alternate paths, however the change is not enough to significantly alter the solution benefit, especially after the fifth allocation.

The locations of these two approaches were also compared and the results arepresented in Table 7. The results with and without the consideration of alternate path travel time are quite different as seen in the Table. Generally, after an examination of network characteristics, the available diversion paths are slightly less congested when including the effects of alternate path travel time. This result is reasonable when considering that the optimal solution may not be unique. That is, there may be many different combinations of solutions that would produce the same objective function value. So while the solutions may be different, the actual end result is the same.

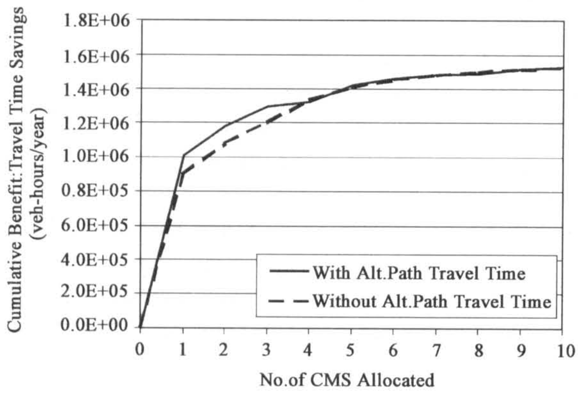

Fig. 12　Cumulative Benefit Comparison: With and Without Consideration of Alternate Path Travel Time

Table 7　Optimal CMS Locations: With and Without Consideration of Alternate Path Travel Time

Allocated CMS at Greedy Iteration #	With Alt Path Time Link ID	With Alt Path Time Link ID
1	7 628	7 628
2	9 980	7 547
3	7 547	9 980
4	9 986	7 615
5	29 486	9 986
6	28 974	$\boxed{28\ 039}$
7	7 615	29 486
8	9 972	$\boxed{25\ 008}$
9	10 250	$\boxed{7\ 559}$
10	7 597	9 972

Conclusions and Future Research

Changeable Message Signs (CMS) are becoming an important component of ITS applications such as Advanced Traffic Management and Traveler Information Systems (ATMS/ATIS). By providing travelers with accurate, timely and reliable traffic information, safety and efficiency of the road network can be improved. The effectiveness of CMS, however, depend on how many CMS are installed and where the CMS are located. This paper presents an optimization model that can be used to systematically locate CMS in an integrated freeway- arterial network. The proposed model consists of four components, including: 1) a multi-period user-equilibrium traffic assignment procedure to estimate traffic volumes on individual links and path flows between individual O-D pairs; 2) a dynamic diversion model that relates the probability for a vehicle to divert, from an incident path to an alternative route with the potential for travel time savings; 3) a time-dependent queuing model to estimate delays with and without the presence of information, and 4) a sequential optimization model to identify the best locations for a given set of CMS.

A sensitivity analysis on the potential impacts of the variations in various parameters on the CMS locations has resulted in the following findings: 1) the optimal locations of CMS are insensitive to variations or estimation errors in traffic demand and incident conditions with the possible exception of large variations in link incident rates; 2) the location algorithm is highly sensitive to the time period considered, however, off-peak periods have little effect on the optimal CMS location; 3) the location algorithm is also highly sensitive to the diversion model parameters.

The model presented in this paper is by no means complete and several modifications could be made in the future to improve the results. First, more research is needed to accurately predict utility model parameters under incident conditions with CMS information. Second, the benefit of other CMS uses (e. g. environmental information) needs to be quantified before they are included in a CMS location model. Lastly, other traffic diversion models that take into account repetition effect of vehicles passing multiple CMS should be considered.

Acknowledgements

This research was supported by the Natural Sciences and Engineering Research Council of Canada.

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